技術系専門試験問題演習講座
記述　熱力学・熱機関

2019年　国家総合職　2次記述　No.16

（熱力学・熱機関）

(1)(2)

(1)(a)

等温変化なので $U=$ 一定.

熱力学I $\Rightarrow Q=W$

$p=\frac{mRT_2}{V}$

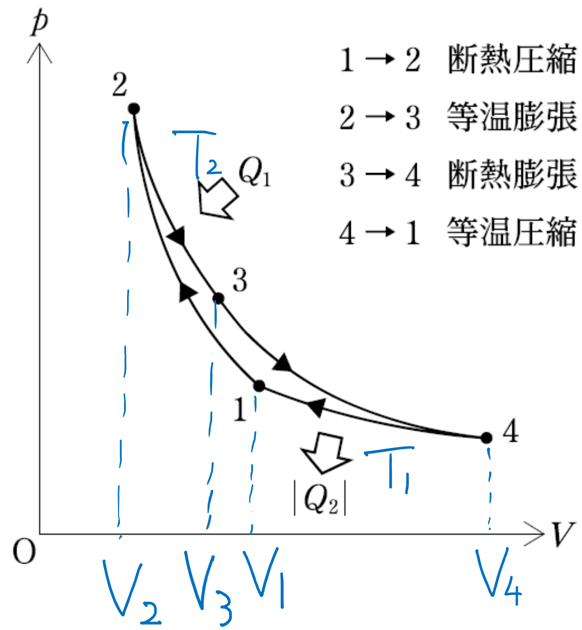
$$Q_1=W_{23}=\int pdV$$

$$=mRT_2\int_{V_2}^{V_3}\frac{dV}{V}$$

$$=mRT_2\log\frac{V_3}{V_2}$$ //

(b)

$$|Q_2|=mRT_1\log\frac{V_4}{V_1}$$



(c) $pV^{\kappa}=$一定の証明

微分形式

① $dU = d'Q - d'W$ (熱力学I)

② $dU = mc_v dT$

③ $d'Q = 0$ (断熱)

④ $d'W = pdV$

c：比熱

$d'Q = mcdT$

$d'Q = TdS$

⑤ $pV = mRT$

$p = \frac{mRT}{V}$

$c_v = \frac{R}{\kappa-1}$

$\therefore\ mc_v dT + pdV = 0$

$pV^{\kappa} = \frac{mRT}{V}V^{\kappa} = (mR)TV^{\kappa-1}$

◎ $TV^{\kappa-1}=$一定を示す。

◎ $\frac{1}{\kappa-1}dT + \frac{T}{V}dV = 0$

$\therefore \int\frac{dT}{T} + (\kappa-1)\int\frac{dV}{V} = 0$ 一定

$\log T + (\kappa-1)\log V = \log TV^{\kappa-1} =$ 一定

$\times V^{\kappa-2}$

$VdT + (\kappa-1)TdV = 0$

$V^{\kappa-1}dT + (\kappa-1)TV^{\kappa-2}dV$

$= d(TV^{\kappa-1}) = 0$

◎ $pV = mRT \Rightarrow Vdp + pdV = mRdT$

$\frac{1}{\kappa-1}(Vdp + pdV) + pdV = 0$

$\therefore\ Vdp + \kappa pdV = 0$

◎ $\left\{\begin{array}{l}\kappa = C_p/C_v \\ C_p - C_v = R\end{array}\right\}\ c_v = \frac{R}{\kappa-1}$

(d) (c)の式より $TV^{\kappa-1}=$一定

$1 \to 2$

$$T_1V_1^{\kappa-1} = T_2V_2^{\kappa-1}$$

$3 \to 4$

$$T_1V_4^{\kappa-1} = T_2V_3^{\kappa-1}$$

辺々わる

$$\frac{V_1^{\kappa-1}}{V_4^{\kappa-1}} = \frac{V_2^{\kappa-1}}{V_3^{\kappa-1}}$$

$$\therefore \frac{V_1}{V_4} = \frac{V_2}{V_3}$$

(e)

$$Q_1 = nRT_2\log\frac{V_4}{V_1}$$

$$|Q_2| = nRT_1\log\frac{V_3}{V_2}$$

$Q_1 : Q_2 = T_1 : T_2$

よって

$$\eta_c = 1-\frac{|Q_2|}{Q_1} = 1-\frac{T_1\log\frac{V_3}{V_2}}{T_2\log\frac{V_4}{V_1}}$$

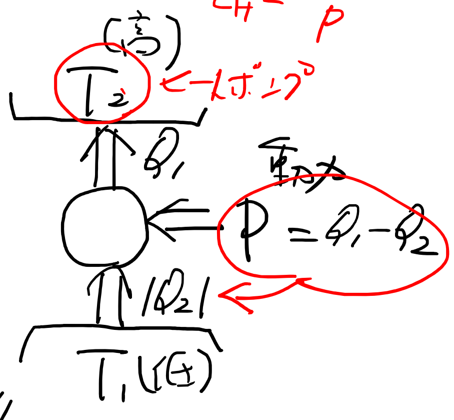
(d)より $\log\frac{V_4}{V_1} = \log\frac{V_3}{V_2}$

$$\therefore \eta_c = 1-\frac{T_1}{T_2} \quad //$$

(f)

$$\varepsilon_r = \frac{|Q_2|}{P} = \frac{Q_2}{Q_1-Q_2} = \frac{T_1}{T_2-T_1} \quad //$$

$$\varepsilon_H = \frac{Q_1}{P}$$



(2)

(a) $1 \to 2$ ($TV^{\kappa-1} = $ 一定)

$$T_1(\varepsilon V_2)^{\kappa-1} = T_2 V_2^{\kappa-1} \quad \therefore T_2 = T_1 \varepsilon^{\kappa-1} \quad //$$

$2 \to 3$ 定圧

$$\frac{T_3}{T_2} = \frac{V_3}{V_2} \Rightarrow T_3 = \sigma T_2 = T_1 \sigma \varepsilon^{\kappa-1} \quad //$$

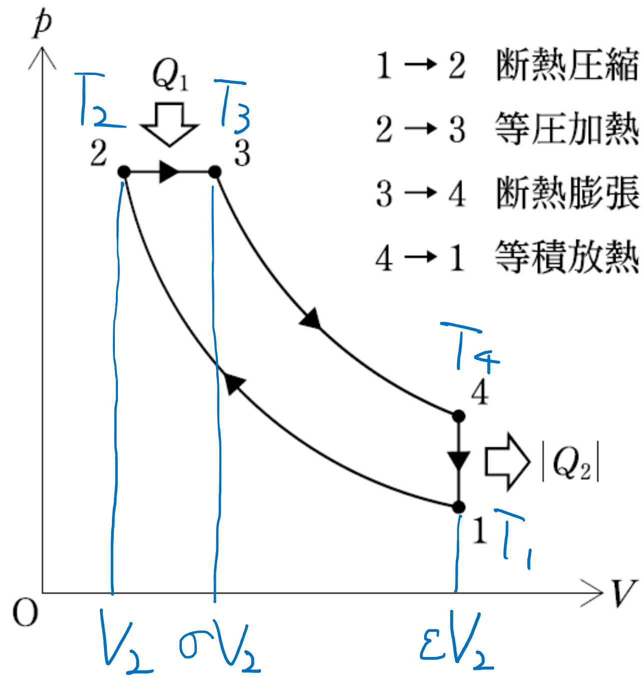
$3 \to 4$ 断熱

$$T_3(\sigma V_2)^{\kappa-1} = T_4(\varepsilon V_2)^{\kappa-1}$$

$$\therefore T_4 = \frac{\sigma^{\kappa-1}}{\varepsilon^{\kappa-1}} T_3 = T_1 \sigma^{\kappa} \quad //$$

(b)
$$Q_1 = mc_p(T_3 - T_2)$$
$$= m\kappa c_v T_1 \varepsilon^{\kappa-1}(\sigma - 1)$$
$$= m\kappa(\sigma-1)\varepsilon^{\kappa-1} c_v T_1 \quad //$$

$$|Q_2| = mc_v(T_4 - T_1)$$
$$= m(\sigma^{\kappa} - 1) c_v T_1 \quad //$$



(c)
$$W = Q_1 - |Q_2|$$
$$= m\{\kappa(\sigma-1)\varepsilon^{\kappa-1} - (\sigma^{\kappa} - 1)\} c_v T_1 \quad //$$

(d)
$$\eta_d = 1 - \frac{|Q_2|}{Q_1} = 1 - \frac{\sigma^{\kappa} - 1}{\varepsilon^{\kappa-1}\kappa(\sigma - 1)} \quad //$$

(e)

$Q_1 = mc_v(T_3 - T_2)$

$Q_2 = mc_v(T_4 - T_1)$

$\therefore \eta_0 = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$

断熱

1 → 2

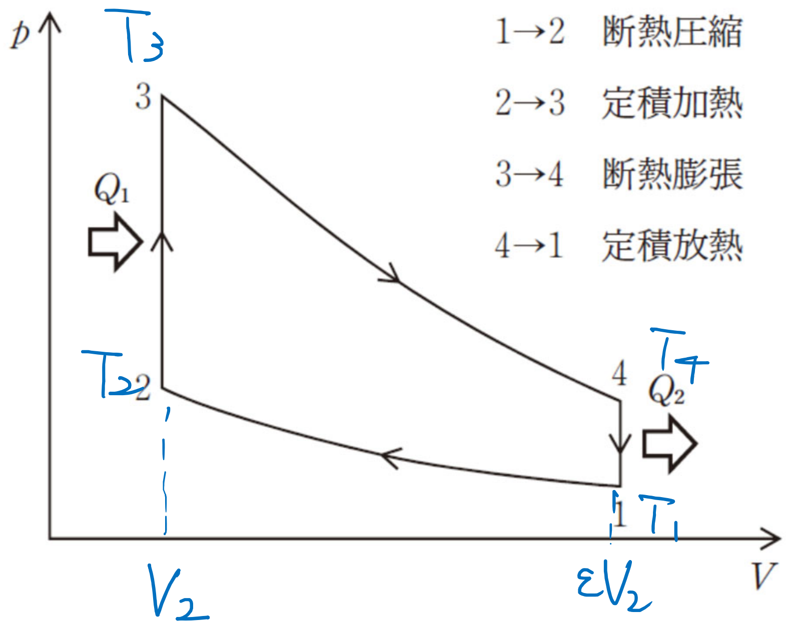
$T_1(\varepsilon V_2)^{\kappa-1} = T_2 V_2^{\kappa-1} \Rightarrow T_1 \varepsilon^{\kappa-1} = T_2$

3 → 4

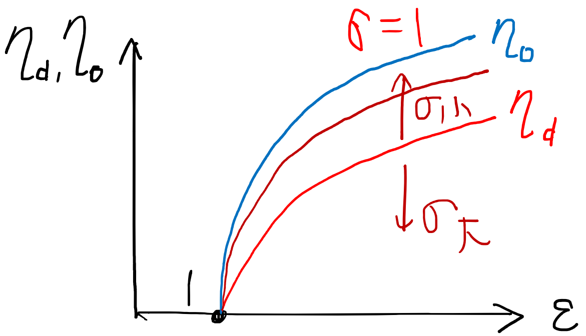
$T_4(\varepsilon V_2)^{\kappa-1} = T_3 V_2^{\kappa-1} \Rightarrow T_4 \varepsilon^{\kappa-1} = T_3$

$\rightarrow (T_4 - T_1)\varepsilon^{\kappa-1} = T_3 - T_2$

$\Rightarrow \eta_0 = 1 - \frac{1}{\varepsilon^{\kappa-1}}$ //



(f)



$$\eta_d = 1 - \frac{1}{\varepsilon^{k-1}} \times \frac{\sigma^k - 1}{\underline{k(\sigma-1)}}$$

⑦ $f(\sigma) = \dfrac{\sigma^k - 1}{\sigma - 1}$ $\quad k > 1$, $\sigma > 1$

$$\lim_{\sigma \to 1} \frac{\sigma^k - 1}{\sigma - 1} = \lim_{\sigma \to 1} \frac{k\sigma^{k-1}}{1} = k$$

ロピタル

$f(\sigma)$ 増加

$$f'(\sigma) = \frac{k\sigma^{k-1}(\sigma-1) - (\sigma^k - 1)}{(\sigma-1)^2} > 0$$

$$= \frac{(k-1)\sigma^k - k\sigma^{k-1} + 1}{(\sigma-1)^2}$$

$/\!/ \; g(\sigma) > 0$

$$g(\sigma) = (k-1)\sigma^k - k\sigma^{k-1} + 1$$

$$g(1) = 0$$

$\nearrow g(\sigma)$ 単調増加

$g(\sigma) > 0 \quad (\sigma > 1)$

$$g'(\sigma) = k(k-1)\sigma^{k-1} - k(k-1)\sigma^{k-2}$$

$$= k(k-1)\sigma^{k-2}(\sigma - 1) > 0 \quad (\sigma > 1)$$

$f(\sigma) > k \quad \therefore \dfrac{\sigma^k - 1}{k(\sigma-1)} > 1$

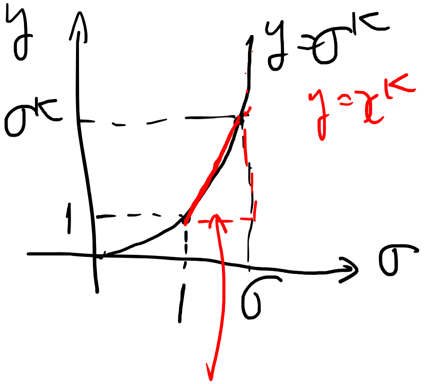
$$\eta_d = 1 - \frac{1}{\varepsilon^{k-1}} \frac{\sigma^k - 1}{k(\sigma-1)} < 1 - \frac{1}{\varepsilon^{k-1}} = \eta_0$$

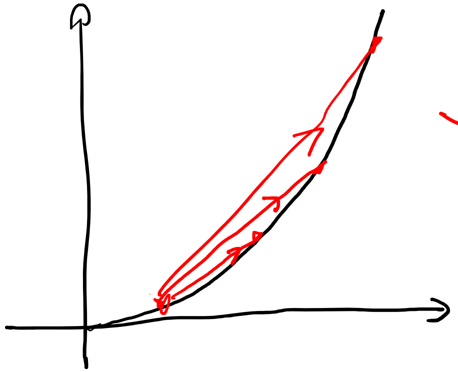
η_d は σ の減少関数

$f(\sigma) = \dfrac{\sigma^k - 1}{\sigma - 1}$ が $\sigma > 1$ で単調増加

$y = \sigma^k$

$k > 1$





傾き

→ $f(\sigma)$は単調増加