

技術系専門試験問題演習講座 総合職 工学の基礎

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(物理)

(解法3)

$$2\pi \sqrt{\frac{m_A}{m_A k'}} = \frac{2\pi}{\sqrt{k'}}$$

$$T = 2\pi \sqrt{\frac{m_A}{k_A}} = 2\pi \sqrt{\frac{m_B}{k_B}}$$

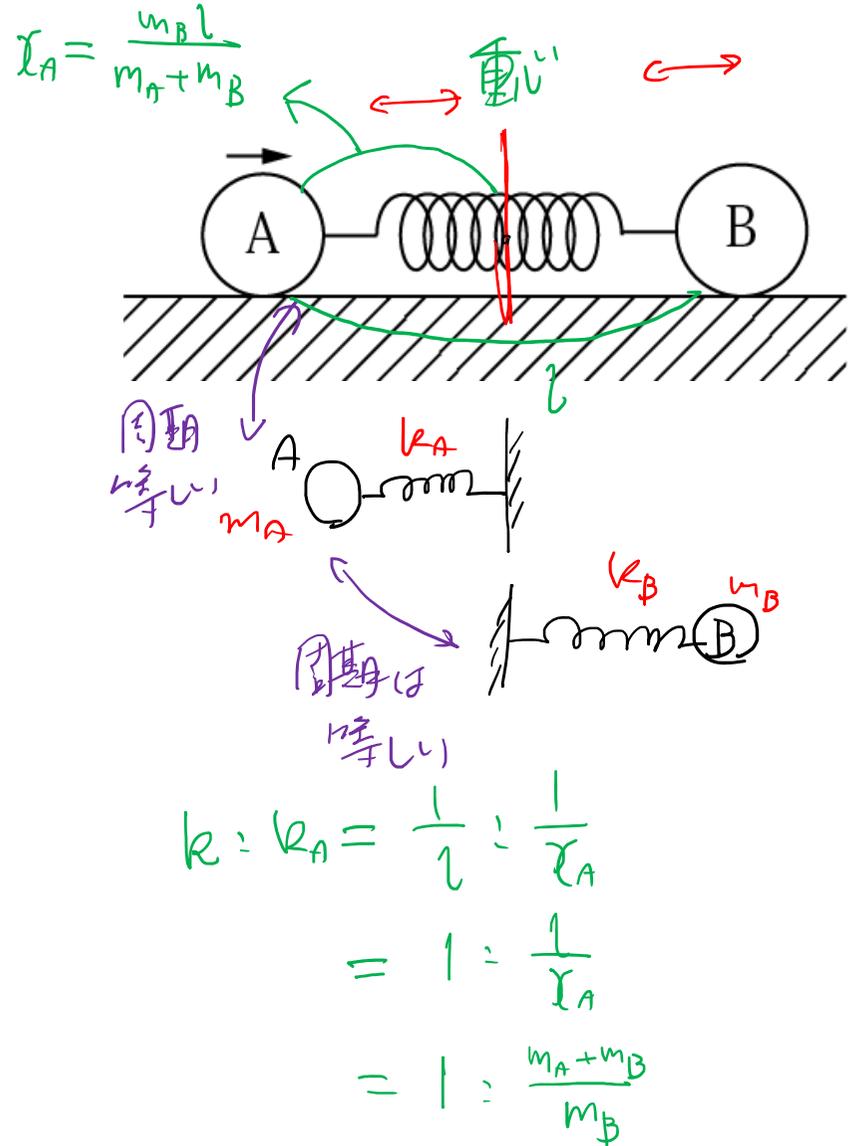
$$\therefore k_A = k_B = m_A : m_B$$

$$\begin{cases} k_A = m_A k' \\ k_B = m_B k' \end{cases}$$

<合流点>

$$\begin{aligned} \frac{1}{k} &= \frac{1}{k_A} + \frac{1}{k_B} = \frac{1}{m_A k'} + \frac{1}{m_B k'} \\ &= \frac{m_A + m_B}{m_A m_B} \times \frac{1}{k'} \end{aligned}$$

$$\therefore k' = \frac{m_A + m_B}{m_A m_B} k$$

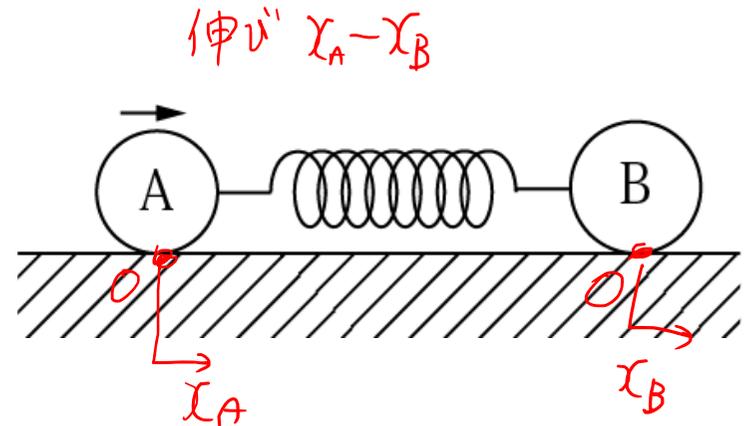


(解4) 運動方程式

$$m_A a_A = -k(\underline{x_A - x_B})$$

$$m_B a_B = -k(\underline{x_B - x_A})$$

減振動



$$a_A = -\frac{k}{m_A}(x_A - x_B)$$

$$a_B = \frac{k}{m_B}(x_A - x_B)$$

$$a_A - a_B = -\left(\frac{k}{m_A} + \frac{k}{m_B}\right)(x_A - x_B)$$

$$a = -\omega^2 x$$

↓ 解法

$$\omega = \sqrt{\frac{k}{m_A} + \frac{k}{m_B}} = \sqrt{\frac{(m_A + m_B)k}{m_A m_B}}$$

$$T = \frac{2\pi}{\omega}$$

(解法) 固有値

$$a_A = -\frac{k}{m_A} x_A + \frac{k}{m_A} x_B$$

$$a_B = \frac{k}{m_B} x_A - \frac{k}{m_B} x_B$$

$$\begin{pmatrix} a_A \\ a_B \end{pmatrix} = - \begin{pmatrix} \frac{k}{m_A} & -\frac{k}{m_A} \\ -\frac{k}{m_B} & \frac{k}{m_B} \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix}$$

$$Q = - \omega^2 x$$

<固有値>

$$A \vec{x} = \lambda \vec{x} \quad (\vec{x} \neq \vec{0})$$

行列 固有値 固有値

$$-A \begin{pmatrix} x_A \\ x_B \end{pmatrix} = -\lambda \begin{pmatrix} x_A \\ x_B \end{pmatrix}$$

$$\lambda = \omega^2$$

$$\omega = \sqrt{\lambda}$$

<固有方程式>

$$|A - \lambda E| = 0$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \rightarrow \lambda^2 - (a+d)\lambda + ad - bc = 0$$

$$\lambda^2 - \left(\frac{k}{m_A} + \frac{k}{m_B}\right) \lambda = \lambda \left(\lambda - \left(\frac{k}{m_A} + \frac{k}{m_B}\right)\right) = 0$$

$$\lambda = \omega^2 = \sqrt{\frac{m_A + m_B}{m_A m_B} k}$$

