



工学系
公務員試験
専門試験問題
演習講座

H29 労基B No.13

数学

実数 x, y が $x+y=2, xy=-1$ を満たすとき, x^5+y^5 はいくらか。

1 33 2 37 3 72 4 82 5 84

$$\textcircled{a} \quad y = \frac{-1}{x} \Rightarrow x - \frac{1}{x} = 2$$
$$\therefore x^2 - 2x - 1 = 0$$

解と係数の関係

$x^2 + ax + b = 0$
の2解を α, β とすると

$$\begin{cases} \alpha + \beta = -a \\ \alpha\beta = b \end{cases}$$

$\Rightarrow x, y$ は $t^2 - 2t - 1 = 0$ の2解

$$t = 1 \pm \sqrt{2} \Rightarrow (1 + \sqrt{2})^5 + (1 - \sqrt{2})^5$$

⑨ 対称式

$$x^2 + y^2 = (x+y)^2 - 2xy$$
$$= 2^2 - 2 \cdot (-1) = 6$$

$$(x^2 + y^2)(x+y) = x^3 + y^3 + x^2y + xy^2$$
$$= x^3 + y^3 + xy(x+y)$$

$$\therefore x^3 + y^3 = (x^2 + y^2)(x+y) - xy(x+y)$$
$$= \underbrace{(x+y)}_2 \underbrace{(x^2 + y^2 - xy)}_{6 - (-1)} = 14$$

$$(x^2 + y^2)(x^3 + y^3) = x^5 + y^5 + x^2y^3 + x^3y^2$$
$$x^5 + y^5 = \underbrace{(x^2 + y^2)}_6 \underbrace{(x^3 + y^3)}_{14} - \underbrace{x^2y^2}_1 \underbrace{(x+y)}_2$$
$$= 84 - 2 = 82$$

⑨ 次数下「 t 」

$$t^2 = 2t + 1$$

$$t^3 = t^2 \cdot t = 2t^2 + t$$

$$= 2(2t+1) + t$$

$$= 5t + 2$$

$$t^5 = t^2 \cdot t^3 = (2t+1)(5t+2)$$

$$= 10t^2 + 9t + 2$$

$$= 10(2t+1) + 9t + 2$$

$$= 29t + 12$$

$$x^5 = 29x + 12$$

$$+ y^5 = 29y + 12$$

$$\underline{x^5 + y^5 = 29(x+y) + 24 = 58 + 24 = 82,}$$